Theoretical Physics Fundamentals
&
Hypercomplex, Thermal, Quantum Fields and Gravitation

This book is dedicated to those that patiently supported me over the years, including my mother, Sybil Morrison, my brothers Eric Morrison and Joshua Morrison, my sister, Teresa Langgaard, and my wife, Cynthia Terry.

Preface

This is intended to be a simple and accessible book on theoretical physics, with topics ranging from introductory physics to advanced theoretical physics. This book draws from undergraduate and graduate physics studies at Caltech, graduate mathematical physics studies at the Maths Institute, Oxford, and graduate physics studies at the University of Wisconsin – Milwaukee. This book begins with the theoretical descriptions of the core physics: (i) Lagrangians and Hamiltonians in classical mechanics; (ii) variational field dynamics – electromagnetism and general relativity; (iii) quantum mechanics and quantum field theory; and (iv) thermodynamics, statistical mechanics, and phenomenology. Then advanced theoretical descriptions are explored in three settings (some of this material is described in four Phys. Rev. D publications, see the Introduction for further details). Those setting are: (1) quantum field theory in curved spacetime, with attention to the the nature of the vacuum; (2) minisuperspace quantum gravity explorations with attention to spherical dust shell collapse; and (3) black hole thermodynamics.

This book can be used as a textbook for four one-semester introductory graduate physics courses based on the material (i)-(iv) above. This book also provides the material for three one-semester advanced graduate physics courses based on the material in (1)-(3) in the above. Exercises are provided at the end of each chapter and detailed derivations are provided for many of the solved examples. The concluding chapter of this book tries to take in the various theoretical underpinnings of physics and arrive at a notion of where things are going next. The future direction proposed influences the selection of 'tangential' topics discussed in the early chapters, so short introduction on the hypothesis is described next.

In *Ch. 9 Hypercomplex Thermal Quantum Gravity*, the central role of path integrals and of time Euclideanization is posited as fundamental in and of itself. The core hypothesis is that complex wavefunctions can be written in a path integral formalism with propagators that involve fields based on

Cayley algebras at all orders. The Cayley algebras with no zero divisors include the Real Numbers (R), the Complex Numbers (C), the Quaternions (denotd H by tradition, since discovered by Hamilton), and the Octonians (O – discovered by Cayley, along with the higher-order 'Cayley algebras'). The stationary phase of a solution, or highly peaked density of states in the Euclideanized time domain, is not possible for fields over the Cayley algebra's that have zero divisors. The zero divisors are posited to disrupt all such higher order Cayley field propagations, thereby eliminating them from path integral considerations except when such a short step is taken that the likelihood of a disruptive (to phase or cohesion) zero divisor occurance is low. This would be inside the Planck time. The deFinetti restriction to complex time propagator may be relaxed as well at such short timescales. Thus, it is hypothesized that the Cayley algebra zero divisors (described in Ch. 9 and the Appendix) appear to play a critical role in understanding how this hypercomplex formalism reduces to the RCHO subalgebra's (that have no zero divisors) that can be used to represent the SU(3)xSU(2)xU(1) Standard Model.

Thus, the path integral formalism appears to be fundamental in many respects, and when coupled with complex wavefunctions made from hypercomplex fields, the path integral formalism appears to offer a number of simplifications in describing the physical model.

The underlying Cayley algebra, that mostly cancels-out in the path integral (or has comparatively lower density of states) to yield an RCHO sub-algebra, may indicate how, in the multidimensional imaginary (thermal) time, the time-reversal invariance is broken and an arrow of time established. One explicit mechanism giving rise to loss of time-reversal invariance is the non-unique inverse that would be allowed if working with a semigroup, and such a construct appears in what follows.

The direct sum decomposition of a Cayley algebra in the neighborhood of an element of the algebra is always quaternionic to lowest order, further suggestive of the local H-algebra representation to directly describe the SU(2)xU(1) part of the model, with similar constructs for the sub-algebras in the Octonians giving rise to the SU(3) part of the model.

Euclideanization, via analytic continuation in the time-domain, takes advantage of a relation that is known to exist between the path-integral formalism (with Action variation in quantum mechanics) and the Weiner Path Integral formalism in statistical mechanics. Evidently time is multidimensional in the two-dimensional complex number sense, why not more, and have the time parameterization generalize via Cayley algebra's

in their own right? The answer might be that they do, but that once again we have a path, or state-density, suppression due a break in a critical multiplicative coherence relation, in this case the de Finetti relation (where there are caustics in information flow if the multiplicative propagators are not complex valued). This suppression of hypercomplex time allows a much more structured, complex analytic, modeling environment to be explored, one where the de Finetti relations are satisfied, and Bayesian statstics is thereby operational. Furthermore, analytic continuation does not exist in the same general sense in the quaternionic and higher algebras. Analytic breaks and de Finetti breaks in the propagation of Feynman type and Weiner type path integral representations, thus, may explain why time should have complex number attributes only. For the spatial components, there is not the unsettling complications found in the hypercomplex time case either (regarding interpretation of 'time'), in the sense that it could be 'whatever works' insofar as representing elementary particles and their interactions.

The concluding chapter, thus, introduces the notion that an infinite, or higher order, Cayley field theory may reduce to an RCHO field theory due to path-integral (or density of states) properties alone. Also note that in the RCHO Theory we may have a 'semiquantum' regime at the Planck length. In efficiently realizing the 'imaginary' confinement that must be enforced in the theory, an inequality constraint may be introduced in the Lagrangian theory. In the appendix the SVM method is described for solving Lagrangians with inequality constraints. A universal geometric algebra, thus, may underlie the physical description of reality, with computationally efficient processes when allowed to have tabular memory during computation, possibly indicating that NP=P (from complexity theory) by its very existence. Realizing NP=P optimality in practice, however, would probably be as difficult as recovering the dispersed, or Cayley-encoded, information content from a black hole radiative extinction event.

Stephen Winters-Hilt

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Chapter 1

Introduction

This book describes physics, the science of the fundamental nature of reality and its constructs. A description of the first half of the book is given in Sec. 1.A on Introductory Physics – Theory and Mathematics. In Sec. 1.B is given an overview of most of the rest of the book, where problems are shown with trying to pin down the role of time. The last section, 1.C, describes the overall conclusion of the book, that there are hypercomplex representations of solutions that may resolve many of the problems encountered in trying to have a unified physics theory.

1.A Introductory Physics – Theory and Mathematics
The fundamental variational calculus tools used in physics are introduced in *Ch. 2 Variational Calculus – Finding the Optimum*, where Classical Mechanics methods are described, including Lagrangian and Hamiltonian formulations, least action, oscillations, special relativity, and introductory general relativity (in a non-geometric tensor calculus setting). Ch. 2 also describes information theory constructs in a variational and geometric context, where a dual geometric construct naturally appears, the possible significance of which is discussed in Ch. 9.

Variational methods must operate with some parameterized set of models or representations to 'vary' over. Three dimensional space, for example, has a number of natural 'flux' (or flow) constructs in terms of vector calculus. The source/sink (div) and rotational (curl) aspects of fluxes and flows are fundamental components even in the earliest formulations of electromagnetic fields. It turns out the the div and curl components in variational calculus are naturally occurring constructs when working with quaternion numbers and not real numbers. The original formulation by Maxwell of his equations were with reference to quaternions in this respect, and it was only when the equations made it across the Atlantic into the hands of American Physicists that the quaternion references were

dropped and the familiar vector calculus definitions introduced (invented) to use in place of the quaternion-based (multiplicative) formulation. In *Ch. 3 Flux and Geometrodynamics*, the description begins with the classical (vector calculus based) electrodynamics theory, followed by a brief description, in modern notation, of the Maxwell equation in quaternionic form. Quaternion invariances in the theory are the familiar Lorentz Invariances in Special Relativity, where the speed of light and transverse wave invariances result as before. Although the quaternionic formulation may be more fundamental in some ways, vector calculus provided the first step towards arriving at tensor calculus, where invariant flux evaluations across specified surfaces appear as contracted tensor calculus terms, which leads to a generalization that allows for a tensor calculus geometrodynamics description. Ch. 3 concludes with a description of Einstein's equations and General Relativity.

A number of experiments in the early 1900's pointed to problems with the classical physics descriptions in terms of real numbers. Although early physicists and electrical engineers already used complex numbers to keep track of phase information in describing wave phenomenon, this was typically viewed as more artifice (mathematical convenience) than reality in some fundamental way. In *Ch. 4 Algebraic Reality – the advent of Quantum Mechanics*, the early (standard) derivations and descriptions of quantum mechanics are given, followed by extensive details on the path integral formalism (that will be central to discussions in Ch. 9). The quantum mechanical description reduces to the classical description as appropriate (in the path integral formalism, as well as in the Schrodinger and Heisenberg formulations), but now a new fundamental structure is indicated: an analytic complex probability amplitude. Once an analytic complex probability amplitude is posited a number of things happen automatically: (1) wave phenomena, such as interference, are now trivially explained in terms of a fundamental, underlying complex probability amplitude in the theory; (2) the wave-aspect of theory has well-known localization limits from the Fourier Transform Uncertainty Principle on transform pairs, that directly translate to both the kinematic and dynamic versions of the Heisenberg uncertainty relations. The quantization of possible bound solutions (standing wave modes), and the quantization of operators generating transformations on compact spaces (angular momentum operators), then result with distinctive effects (the Stern – Gerlach experiment, etc.), as discussed in Ch. 4.

Observation in quantum mechanics typically interferes with what is being observed (limits on quantum non-demolition are discussed in Ch. 4). Early discoveries in quantum mechanics were confounded by two sets of

mysteries, one having to do with the representation of reality itself, and one having to do with the representation of the measurement process that is established by the experiment. These matters are discussed briefly in Ch. 4, as well as a description of relativistic quantum mechanics (Dirac).

Once moving to a relativistic theory a number of interesting mathematical objects join the toolkit, including spinors and Dirac matrices. It will be shown that generalizations to quantum field theory (Ch. 6) will give rise to even more mathematical objects, such as the Gell-Mann matrics in color SU(3) gauge field theory. It turns out, however, that each of these esoteric mathemativcal inventions, particularly the Dirac and Gell-Mann matrices, and their equations, can be expressed directly in terms of quaternionic and octonionic algebra formulations of the theory (just as esoteric perhaps, but we are now talking about the fundamental Cayley algebra's), where there is only the multiplicative operation of the respective Cayley algebra, and no need for matrix constructs, or vector calculus operations.

The discovery of the fundamentally algebraic nature of reality would have been quicker if there was more universal acceptance of the strangely appearing complex probability amplitude construct central to the theory. Having a differential equation acting on a complex function, the notion of analyticity naturally arises, and it is at this juncture that it is instructive to understand how close the early quantum mechanics theorists were to establishing that a propagator-based theory would require such a complex probability amplitude. We begin with the Diffusion equation, a fundamental motion found for random walks, among other things. (Markov chains and Martingales are briefly reviewed in Ch. 4 to put the fundamental nature of the Diffusion equation into context.) By considering the time parameter to be analytic, and by performaing analytic continuation to complex time, the Schrodinger equation results. The real diffusion equation acts on a real density function (with a measure, so equivalent to a probability, e.g., this gives rise to standard probability theory and statistics). The complex Schrodinger equation acts on a complex wavefunction, interpreted as a probability amplitude, not a probability itself. Just like the Diffusion equation has a propagator formalism, so does the Schrodinger equation. By the deFinetti relation, however, for a propagator-based theory to be completely multiplicative, the propagator must be complex valued. A complex, analytic, propagator formalism, and wavefunction description, thereby is indicated and quantum mechanics results.

The fundamental existence of Equilibrium and Flow phenomena in physics can be related to the fundamental existence of the Law of Large

numbers, the Asymptotic Equipartition Property (AEP), and the Martingale Limit Theorems in stochastic analysis. In Thermodynamics, the central function describing the system is the partition function. In Statistical Mechanics the partition function is part of a larger construct, in terms of a density of states and path integral formulation. These topics are discussed in *Ch. 5 Thermodynamics, Statistical Mechanics, & Phenomenology*, where detail is first given to the derivation of the fundamental laws of thermodynamics. When cast in a path integral formalism, with a multiplicative propagator, the theory is directly transformable, via rotation to complex time, to a quantum or quantum field theory path integral formalism (the latter described in Ch. 6). The complex time rotation, or Euclideanization of a quantum mechanical path integral to arrive at a statistical mechanics path integral, is actually a central method to making the quantum path integral well-defined (see Ch. 4). The extensive use of complex time manipulations in thermal quantum field theory (see Ch. 6) lends further credence that there is some meaning to complex time beyond the convenience of it's compact representation for both quantum mechanical and statistsical mechanical attributes of a system.

1.B Quantum field theory and general relativity – the role of time

When quantum mechanics and special relativity are combined, quantum field theory results. Just describing the simplest field theories and their scattering results can be revealing about the nature of reality. In *Ch. 6 The Quantum Vacuum and Perturbative Reality*, it is shown that even with the simplest field theory, nothing!, the quantum vacuum observed is related to the observer's trajectory (with causal horizon effects) and local time sense. In curved spacetimes where horizon effects occur, there may exist no trajectory free of particle production effects (so you can't have nothing). So our motion (i.e., trajectory) effects the representation of the measurement device (mentioned earlier in regards to quantum measurement). The maximal Fermi Normal Coordinate (FNC) parameterization (foliation) of the observer's neighborhood along their trajectory must map information flow at boundary via a non-trivial Bogoliubov transformation, so have particle influx from the maximal FNC boundary -- unless the FNC region extends throughout the spacetime and there is no boundary.

The immense success of quantum field theory was established with the renormalization of QED via the Feynman Path Integral approach to arrive at a highly accurate perturbation theory result. This success also strengthens the notions of thinking of reality in terms of perturbation theory algebraic constructs – e.g., semigroups. Quantum field theory also offers a string of other successes in describing causal horizon particle

production effects, such as with Hawking radiation. In Ch. 6 these matters are explored and a critical dependence on "choice" of time in defining the quantum vacuum is revealed.

So, we've come all the way to QFT to find no answers regarding time, only further representational complexities and mysteries regarding the nature of time. Time appears to be related to your local notion of 'nothingness' or 'unchangingness', and to your local trajectory as observer (see Ch. 6). Even if a choice of time might be uniquely specified by a patchwork of FNC spatial slices, with non-trivial Bogoliubov transform particle fluxes specified at mesh regions (where one FNC patch overlaps and continues with another FNC patch), this approach does not seem to offer further insight into the nature of time.

In order to consider a non-field-theory situation, where symmetries have eliminated all but a few of the degrees of freedon, and where a full general relativity solution is still possible, spherical dust shell collapse is described in *Ch. 7 Geometry and Action*. Again, however, the nature of time remains elusive, and it is found that the choice of time in the collapse quantization scheme critically impacts the spectrum of observations.

In *Ch. 8 Thermal Geometrodynamics*, the path integral formulation of a black hole (BH) is described, and using an analytic function of time, the Hawking-Gibbons form of BH entropy is obtained by using Euclideanization. The Hamiltonian Thermodynamics of some BH systems are then described, and an attempt is made to understand the role of time in this context. The key metric parameters of the BH theory obey fall-off conditions at asymptotic boundary regions. This represents a fundamental semigroup parameterization of the perturbatively stable (metric) parameters of the BH theory. This is an entirely separate appearance of perturbatively stable behavior from that appearing in the QED renormalization via Feynman path integrals described in Ch. 4 and Ch. 6. In thermal geometrodynamics, as with thermal quantum field theory, the success of Euclideanization is profound.

1.C Universal hypothesis of number, computation, and physics
Reality is known to be variationally optimal, perturbatively stable, algebraic in a variety of ways, and has thermality via Euclideanizable (complex) time. The variational optimization itself can result from selection for stationary phase in a path integral description, where the fundamental construct is the propagator.

In Ch. 9 a Universal Hypothesis is provided for describing physical reality. The hypothesis builds from a collection of constructs known to be useful, but not typically construed as fundamental. The pillars of the universal hypothesis are as follows:

(1) Reality is described in terms of a completely multiplicative propagator.

(2) The path integral formalism resuling from the propagotor has two forms, Feynamn Path Integral and Weiner Path Integral, according to the introduction of an analytic (complex) time parameter, where the forms are related via Euclideanization.

(3) The Path Integral formalism selects against fields involving algebras with zero divisors, thereby reducing to a RCHO(S) based theory, where equilibrium and martingale constructs occur asymptotically no matter the initial condition.

(4) The algebraic field propagated, from one field algebra to the next, either has a shared quaternionic sub-algebra, or evolves towards a common quaternionic sub-algebra by means of a maximum divergence step.

In order to have a completely multiplicative description the propagator must be complex valued. This can be proved by considering how many real parameters $f(n)$ are needed to specify an n-dimensional mixed state. By mixed state we essentially mean matrix representation, symmetric, with a real diagonal (where the symmetry operation is conjugation for the Cayley algebras). For the mixed state to be parameterized by real values there are $[n+n(n-1)/2]$ parameters (denote Cayley order for reals as $k=1$, for complex, $k=2$, for quaternions, $k=3$, etc.). The number of real values in the self-conjugate k-th order Cayley algebra mixed state is $f(n) = kn(n-1)/2+n$, and we only get $f(n)=n^2$ when $k=2$. The importance of having $f(n)=n^2$ is that now have $f(n_a n_b)=f(n_a)f(n_b)$, which results in a 'completely multiplicative' theory, thus, the propagator must be complex valued.

In *Ch. 9 Hypercomplex Thermal Quantum Gravity*, the central role of path integrals and of time Euclideanization is posited as fundamental in and of itself. The core hypothesis is that complex wavefunctions can be written in a path integral formalism with propagators that involve fields based on Cayley algebras at all orders. The Cayley algebras with no zero divisors include the Real Numbers (R), the Complex Numbers (C), the Quaternions (denotd H by tradition, since discovered by Hamilton), and the Octonians (O – discovered by Cayley, along with the higher-order 'Cayley algebras'). The stationary phase of a solution, or highly peaked density of states in the Euclideanized time domain, is not possible for fields over the Cayley algebra's that have zero divisors. The zero divisors

are posited to disrupt all such higher order Cayley field propagations, thereby eliminating them from path integral considerations except when such a short step is taken that the likelihood of a disruptive (to phase or cohesion) zero divisor occurance is low. This would be inside the Planck time. The deFinetti restriction to complex time propagator may be relaxed as well at such short timescales. Thus, it is hypothesized that the Cayley algebra zero divisors (described in Ch. 9 and the Appendix) appear to play a critical role in understanding how this hypercomplex formalism reduces to the RCHO subalgebra's (that have no zero divisors) that can be used to represent the SU(3)xSU(2)xU(1) Standard Model.

Thus, the path integral formalism appears to be fundamental in many respects, and when coupled with complex wavefunctions made from hypercomplex fields, the path integral formalism appears to offer a number of simplifications in describing the physical model. In brief, the idea is that paths over the Sedenions (S) and higher-order Cayley algebras have zero divisors, so can have their phase information lost, or scrambled, in path intregral contributions, and thereby eliminated when summed. This would 'disappear' algebraic constructs in the theory with zero divisors (and very quickly if a quantum evolution 'step' is on the order of a Planck length). The lowest Cayley algebras, the RCHO algebras are, thus, considered critical to the field descriptions in this hypothesis (called the RCHO hypothesis in what follows). As will be shown, SU(2) can be represented using quaternions (SU(2) is isomorphic to quaternions of absolute value 1) and SU(3) can be represented as the sub-group of octonian automorphisms leaving a given imaginary unit invariant. If we restrict to matter fields with SU(3) derived by such automorphisms on the quaternion field's three imaginary values, we arrive at the three matter generations, as observed. In this context, the emergent RCHO matter field phenomenologies have no zero-divisors, so have inverses, so describe the usual time-reversal invariant theories (and thereby introduce the usual entropic paradox).

The direct sum decomposition of a Cayley algebra in the neighborhood of an element of the algebra is always quaternionic to lowest order, further suggestive of the local H-algebra representation to directly describe the SU(2)xU(1) part of the model, with similar constructs for the sub-algebras in the Octonians giving rise to the SU(3) part of the model.

Euclideanization, via analytic continuation in the time-domain, takes advantage of a relation that is known to exist between the path-integral formalism (with Action variation in quantum mechanics) and the Weiner Path Integral formalism in statistical mechanics. Evidently time is

multidimensional in the two-dimensional complex number sense, why not more, and have the time parameterization generalize via Cayley algebra's in their own right? The answer might be that they do, but that once again we have a path, or state-density, suppression due a break in a critical multiplicative coherence relation, in this case the de Finetti relation (where there are caustics in information flow if the multiplicative propagators are not complex valued). This suppression of hypercomplex time allows a much more structured, complex analytic, modeling environment to be explored, one where the de Finetti relations are satisfied, and Bayesian statstics is thereby operational. Furthermore, analytic continuation does not exist in the same general sense in the quaternionic and higher algebras. Analytic breaks and de Finetti breaks in the propagation of Feynman type and Weiner type path integral representations, thus, may explain why time should have complex number attributes only. For the spatial components, there is not the unsettling complications found in the hypercomplex time case (regarding interpretation of 'time'), in the sense that it could be 'whatever works' insofar as representing elementary particles and their interactions.

The Feynman-Cayley-Shannon Information Hypothesis
The central hypothesis that results is that we have maximal Euclideanizable propagation, or flow, of Feynman-Cayley-Shannon Information (see Ch. 9). The first primal physical construct in this approach is a multiplicatively complete propagator that is Euclideanizable, such that a Feynman Path Integral (FPI) approach results in real time and the Weiner Path Integral (WPI) results in the FPI imaginary time analytic continuation. The well-defined Weiner Path Integral provides a means to indirectly evaluate the Feynaman Path Integral, and in that manner make it well-defined also. The flow of information is now summarized in terms of stationary phase solutions, for FPI, or in terms of highly-peaked density of states, for WPI. The core elements, the little steps used to build the paths, are traversed by use of propagators. In Ch. 9 the central ESCK relations are given, and how in the context of a random walk they automatically reduce to the diffusion equation. In the larger sense of Markov processes, Markov Chains (MCs), and hidden Markov models (HMMs), it is found that Martingales place a central role (Ch. 9 provides details). MCs induce martingales by themselves alone, while the core algorithms in HMM analysis involve dynamic programming tables (tabular memory usage) in a log-likelihood ratio evaluations (a tabular multiple martingale construct).

The appearance of martingales in the core propagator construct isn't surprising given the role of each in describing evolution by repeated steps,

but the fundamental aspect of martingales, in describing equilibrium and stationarity, is not generally well-known. Martingales have convergence and limit properties like those encountered for the strong law of large numbers, the backward martingale convergence theorem directly leading to a the strong law of large numbers as well as the asymptotic equipartition (AE) property. The AE property, in turn, is fundamental to both thermodynamics and statistical mechanics, where equilibrium states are proposed that satisfy the AE property, likewise with the Path Integral approaches, where the AE approach allows all paths to simply be "summed" and not a weighted sum with some unknown set of weights. The AE Property originated with the work of Shannon, who was applying it to problems in signal processing, communication, and information theory. The multiplicatively complete propagator that is Euclideanizable is, thus, hypothesized to induce martingales in a variety of forms with the equilibrium, near-equilibrium stationarity (flow), and AE properties emerging as a result. The stationary phase solution in the FPI approach leads to classical physics solutions and semiclassical quantum mechanics solutions. In the WPI approach, the natural occurance of martingales is effectively a stationary expectation solution, where stationary expectations are expected to occur in equilibrium.

The first primal physical construct in this approach is a multiplicatively complete propagator (thus complex) that is Euclideanizable and that induces martingales. The second primal physical construct is the field upon which the propagator is defined. If the field theory involves the higher order Cayley algebras above the octonians, then the path integral construct will not contribute as greatly due to the occurrence of zero divisors in the theory (an aspect of irreversibility, e.g., no inverses, in the theory). This is hypothesized to suppress all the highe order Cayley algebra fields that have zero divisors (unless endowed with meaning, such as light cone encodings in the case of the split octonian representation in terms of two quaternions). Thus, the effective quantum field that results is based on the Octonian algebra at highest order as a group with no zero divisors, or is based on the Conway-Smith split-Sedenion algebra that is the highest order for a semigroup with no zero divisors (with only one-sided multiplication). As mentioned previously, reality appears to be perturbatively described at a fundamental level, as shown in the highly successful QED and related theories. Multiplication with perturbation expansion is a semigroup operation and suggests that the split-Sedenions be taken to reach the highest order 'no zero divisor' theory consistant with a perturbative (semigroup) theory.

A number of embedded Cayley algebra's arise in a natural way. Recall that embedded in a Cayley algebra at any order above quaternionic, there is a quaternionic sub-algebra. Similarly, within that quaternionic algebra is a complex sub-algebra. The embeddings on sub-algebras works for octonians within higher order Cayley algebras, also, but beyond that the sub-algebras don't appear to embed – the three element associator symbol inherent to all the Cayley algebras can have two orthogonal doubly pure zero divisors inserted to provide a mapping of all the higher order Cayley algebras to the Octonion sector generated by the associator with the two (sedenion or higher order) zero divisors indicated (so only have a $C \subset H \subset O$ embedding description within the higher order Cayley algebras).

This appears to suggest a Conway-Smith split-Sedenian perturbation field theory construct. Embedded in the split Sedenian semialgebra are two octionic subalgebras, which suggest a representation of stable matter in terms of octionic fields (when non-associative contributions are dropped one arrives at Maxwell's equations). Embedded within the Octonian matter respresentation is a quaternionic spacetime representation. The quaternionic spacetime parameterization automatically provide notions of flow and vector calculus relations such as div and curl in the definition of group multiplication. Within the quaternionic representation is a complex algebra representation, this is the only Cayley algebra that will mesh with the propagator (and wavefunction) restriction to be only complex in the time parameter. So we are talking about a propagator that operates on a sedenion filed and that propagates that field to another sedenain field. In this propagation we know that the reference spacetime changes via some Galilean or Lorentzian shift (depending on approximation needed) if 'in equilibrium', or a maximum divergence shift in the sedenian field otherwise (the third primal construct appears here as the maximum divergence principle, which reduces to the maximum entropy principle. When the field approaches equilibrium, the sedenians can be viewed as sharing the same quaternionic subspace. Thus, the equilibrium theory, or near equilibrium theory, both describe a 16 dim object propagating to another 16 dim object, where a 4 dim subspace is shared. In other words, this is a 16+16-4 =28 theory, or a 26 dimensional theory with a 2 dim object (a string). It may be that string theory arises in this context in a new way. If a link to string theory can be demonstrated, then significant progress would be made to establishing a well-defined theory of quantum gravitation. This is because many of the renormalization difficulties are eliminated in the string formalism.

In the split-sedenion hypothesis, the gravitational degress of freedom are only free to enter via a 'geometric' octonion, to differentiate from the 'matter' octonian mentioned above. The non-associative components resulting from these octionic fields may be reduced in their path integral support due to density of states spreading effects – by dropping the non-associative terms it is shown in [X] that Maxwell's equations can be obtained. As mentioned, when not in equilibrium, the sedenion propagation is to be done according to the maximal divergence. When in this form, where the divergence is nonzero, a dually flat (local quaternionic) descriprion exists, showing the deeper significance of the evolutionary process in the context of the em-algorithm in information geometry [X], (which is the justification for the maximal divergence rule in the third primal construct).

The above approach motivated by Feynaman, Cayley, and Shannon, recovers everything except quantum gravity. If gravitational fields can be represented in terms of octionic algebras, however, this process may become straightforward. Even if successful to that extent, however, eventually a quantum gravity renormalization will run afoul of its use of a dimensionful coupling constant, thereby giving rise to a countable infinity of terms requiring renormalization. The current approach, however, still offers a way out in terms of the infinite number of counter terms that come from suppression of the infinite higher order Cayley algebras. This also might clarify where the break in the underlying time-reversal invariance occurs. Given the possible relation to a description in 26 dim with a 2 dim object (string), the renormalization difficulties may be trivially eliminated by relating this theory to the well-behaved (no point-singularity) 26 dim string theory.

The familiar semiclassical regime is defined according to the wavelength of the object. In the RCHO Theory we may have a 'semiquantum' regime at the Planck length as well. In efficiently realizing the 'imaginary' confinement that must be enforced in the theory (of the higher oder Cayley algebras), an inequality constraint may be introduced in the Lagrangian theory. A universal geometric algebra, thus, may underlie the physical description of reality, where path integral evolution is described in terms of a multiplicative propagator, possibly indicating that NP=P (from complexity theory) by its very existence. Realizing NP=P optimality in practice, however, would probably be as difficult as recovering the dispersed, higher order Cayley-algebra encoded, information content from a black hole radiative extinction event.

<div align="right">Stephen Winters-Hilt</div>

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