Meromorphic precipitation of quantum matter with dimensionful action

Stephen Winters-Hilt

Meta Logos Systems
Albuquerque, New Mexico, USA

Abstract
Efforts to identify a formulation for maximal algebraic information flow have led to the examination of unit-norm trigintaduonion multiplications. In prior work this was considered without the complication of zero divisors. In an effort to remove the zero divisors by requirement of maximum domain of analyticity on the log trigintaduonion multiplication we find a theoretical framework exists for meromorphic precipitation of matter. In this process a fundamental quantum is indicated from the zero divisor residue terms. Analyticity in the form of a Wick rotation also provides a mechanism whereby we can switch to a dimensionful action and quantum.

Introduction

This paper describes the mathematical properties of maximal algebraic information flow. In prior work [1-4] this was considered without the complication of zero divisors. We will see that zero divisors act as "sources", so the prior work was effectively analysis of sourceless information flow. In this paper we consider zero divisors and their impact on the maximal information flow and in doing so see a mechanism for meromorphic precipitation of quantum matter with dimensionful action.

In prior work [1-4] we hypothesized maximal algebraic information flow, where the "emanation" of information is represented as multiplication by an element of an algebra in two steps: (i) take the maximal current-state element that is a unit-norm trigintaduonion; and (ii) perform the emanator step that consists of an achiral sum of multiplications with chiral trigintaduonion emanators.

Consider the second step above. There are 4-types of chiral emanators, with 14 sub-types each, thus there are 4x14=56 emanator subtypes. The non-perturbed chiral emanator is a 10D algebraic element. (For a given chiral propagation, there are then 22D of motion that are frozen with respect to the 10D non-perturbed.) When the maximal perturbation is allowed into the enveloping 32D trigintaduonion algebra a particular choice of chiral propagation will have 29D real dimensions of freedom (29*D effective dimensions when analytic), for which we find maximum perturbation when $\alpha=1/137^*$. A relation involving parameters $\{\alpha,\pi\}$ was derived in [2,4]:

$$\alpha = (1/137)(\cos\beta/\cos\theta)(\sin\theta/\theta), \text{ where } \beta = (\pi/137) \text{ and } \theta = \pi/(137 \times 29).$$

Consider the second step when all emanations are summed and normalized to a unit norm. The normalization sum is achiral on the core subtypes identified, with effective dimension 29*, such that the fractal edge-of-chaos choice for maximal perturbation gives the $\{\alpha,\pi,c_\infty\}$ relation [4]:

$$\alpha^{-1} = (c_\infty)^\gamma\,,\ \gamma = (1/2)(29^*),\ c_\infty = 1.401155189\ldots,$$
$$\text{where } \gamma = (1/2)(29 + (4\pi/72)[(1+\theta\{(\pi/72)+(3/72)\}]),\ \text{and } \theta=\pi/(137\text{x}29).$$

In what follows we will consider trigintaduonion emanation when multiplication can involve the occurrence of zero-divisors. We will see that zero-divisor-removal emanation shall associate with matter propagation (see Results), while zero-divisor achiral emanation shall associate with wave-collapse (see Discussion). Since critical properties of fractal boundaries, zero-divisors, and integrals-with-large-parameter are not well known, they will be reviewed first, in the Background sections that follow, along with a recap of previous results.

Background

Previous Results -- Maximum Information Flow Part I: Fiat Numero

(1) Maximal Information flow without perturbation is in 10D chiral subspace of 32D trigintaduonions [1];

(2) Maximal perturbation is by amount α=1/137* into the enveloping 32D space [2];

(3) Maximal Information flow with perturbation is 32D, where all chiral subtypes are summed and normalized, with the four 29*D chiral propagations possible in the analytically continued 32D complexified space (64D). We thereby arrive at the edge-of-chaos maximal information emanation relation, where $\alpha^{-1} = (c_\infty)^\gamma$, with its suggestion that the universal evolution is at a fractal boundary, and thus fractal itself [4].

Maximum Information Flow Part II: Logos Incarnate

(4) The dimensionless quantum arises from analyticity in the form of a meromorphic function association to each of the 29D in a given chiral propagation, where associated zero divisor (ZD) surgery gives h*<<α<1 since each ZD has 29 real component dimensions (plus a remnant of imaginary dimensionality, thus effective dim=29* [4]), and where a point-like location is given by the location of the cut-out.

(5) In what follows, we shift from emanator projection to discrete-time propagation with (S*/h*) and, most notably, a shift from propagation in terms of trigintaduonion emanation steps comprising trigintaduonion multiplications to the more conventional propagation in terms of complex propagators comprising multiplication of complex functions of a complex variable. The shift from 32D emanator numbers to 2D propagator complex functions is necessitated by consistency with the maximal info flow hypothesis and the known constraints of the quantum deFinetti relation to information flow with complex propagators [4].

(6) The dimensionful quantum arises from Wick rotation from real to pure imaginary (with ZD cut-outs) such that (S*/h*) with discrete time steps 'n' Wick rotates to S/h with dimensionful time 't'. The exact numerical relation h* →|h| may be a truly random emergence that will never be defined further. The main constraint, which is satisfied, is that the quantum be very small, giving rise to an oscillatory integral formalism. A shift in the small constant can't be explained further with the current development of the theory. Experimental data is used to justify the dimensionful choices of time in seconds, etc.

Complex functions, mappings, and fractals (2D → 2D transforms)
The 2D plane can be mapped repeatedly to itself. For such situations the asymptotic behavior can be examined. Take for example the classic Mandelbrot set mapping where

$$z_{new} = f(z_{old}) = (z_{old})^2 + c$$

For the Mandelbrot set the stability boundary has fractal dimension 2 [5]. One might guess a number $1 \leq frac \leq 2$, but to actually reach the dimension 2 shows an optimality for the Mandelbrot set in this regard that will be called upon later where it will lead to a "zero-divisor" occurrence under that circumstance, thereby effecting a second order zero. The order of the zero will be relevant to the initial dimensionless Planck "h*" calculation (when using the residue theorem from complex analysis).

Trigintaduonion mappings and Zero Divisors (32D → 32D transforms)
We describe emanation as going from an old trigintaduonion base to new trigintaduonion base, i.e, a 32D-to-32D transform. For some of the analysis the 32 real component trigintaduonion will decompose into a product of N real dimensions, each analytically continued to have a product contribution from its residue factor, this will be described further in the Results. For some of the analysis the 32D trigintaduonion isn't reducible, however, such as for consideration of zero divisors that are described next.

Zero Divisors
The division algebras do not have zero divisors and comprise the first four algebras of the Cayley family: the real numbers, the complex numbers, the quaternions, and the octonions. Beyond octonions the Cayley algebras have zero divisors. The next two Cayley algebras are the sedenions and the trigintaduonions. Let's begin by analyzing the zero divisors for the sedenions. Consider the situation:

$$S_1 \cdot S_2 = 0, where\ S_1 = (O_{1L}, O_{1R}) \neq 0\ and\ S_2 = (O_{2L}, O_{2R}) \neq 0$$

S_1 and S_2 in the above form a zero-divisor pair. Let's carry the analysis to the level of octonions to extract more manageable relations:

$$(O_{1L}, O_{1R}) \cdot (O_{2L}, O_{2R}) = ([O_{1L} \cdot O_{2L} - O_{2R}{}^* \cdot O_{1R}], [O_{1R} \cdot O_{2L}{}^* + O_{2R} \cdot O_{1L}]).$$

For the last expression to be zero, it must be zero component-wise, and we arrive at two relations:

$$O_{1L} \cdot O_{2L} = O_{2R}{}^* \cdot O_{1R} \quad and \quad O_{1R} \cdot O_{2L}{}^* = -O_{2R} \cdot O_{1L}$$

Let's consider the simplest case, where the four octonions are unit octonions

$$\{O_{1L}, O_{2L}, O_{1R}, O_{2R}\} \in \{e_i\}\ i = 0..7,$$

where

$$e_i \cdot e_j = \begin{cases} e_j \, if \; i = 0 \\ e_i \, if \; j = 0 \\ -\delta_{ij} e_0 + \varepsilon_{ijk} e_k \end{cases}$$

where the antisymmetric tensor is 1 when ijk = $\{123,145,176,246,257,347,365\}$. The conditions are then simplified to (unit octonion for O_{1L} is labeled as e_{1L}):

$$\varepsilon_{(1L)(2L)(k)} = \varepsilon_{(1R)(2R)(k)} \; and \; \varepsilon_{(1R)(2L)(k)} = -\varepsilon_{(1L)(2R)(k)}.$$

Consider the first zero divisor indicated, where:

$$\varepsilon_{(1)(4)(5)} = \varepsilon_{(3)(6)(5)} \rightarrow \{1L = 1, 2L = 4, 1R = 3, 2R = 6\}$$

Thus, $S_1 \cdot S_2 = 0$, with $\{S_1, S_2\}$ is a zero divisor pair, if $S_1 = (e_1, e_3)$ and $S_2 = (e_4, e_6)$. If converted to sedenions (see multiplication table at [6]):

$$S_1 = (e_1, e_3) = (\hat{e}_1 + \hat{e}_{3+8}) = (\hat{e}_1 + \hat{e}_{11})$$

$$S_2 = (e_4, e_6) = (\hat{e}_4 + \hat{e}_{6+8}) = (\hat{e}_4 + \hat{e}_{14})$$

$$S_1 \cdot S_2 = (\hat{e}_1 + \hat{e}_{11}) \cdot (\hat{e}_4 + \hat{e}_{14}) = \hat{e}_5 + \hat{e}_{15} - \hat{e}_{15} - \hat{e}_5 = 0.$$

So we see that there are zero divisors in the sedenions, with a concrete example above, for any set of indices that can be chosen for the antisymmetric tensor in its first two positions. Thus i can take 7 values and j 6 in the relation: $\varepsilon_{ij(k)}$, so 42 cases. A similar set of relations exist for the negative antisymmetric tensor indices for another 42 cases. So there are 84 such zero divisors.

There are only 84 discrete instances of zero divisors for the sedenions. Can this number be increased by relaxing assumptions in our derivation above? (1) Can we interpolate with $S_1 = (\tau e_1, (1 - \tau) e_3)$ for some variety of τ? If we try this the antisymmetric tensor forces the single (equilibrium) case where $\tau = 1/2$. (Norm=1 would force this as well.) (2) Can we generalize solutions of the form $(\hat{e}_1 + \hat{e}_{11}) \cdot (\hat{e}_4 + \hat{e}_{14}) = 0$ to sedenions consisting of more than the addition of two unit sedenions? (In turn, this traces back to assuming the octonion decomposition consisted of unit octonions.) For this to work we would have an expression:

$$\{3 \; \hat{e}_i \; term\} \times \{3 \; \hat{e}_j \; term\} = \{9 \; \hat{e}_j \; term\} \rightarrow odd \; terms, can't \; cancel \; pairwise, no \; zd's$$

trying again with expression with four unit vector terms:

$$\{4 \; \hat{e}_i \; term\} \times \{4 \; \hat{e}_j \; term\} = \{16 \; \hat{e}_j \; term\} \rightarrow need \; 4 + 4 + 8 = 16 \; indep. imag's$$

For the latter case, we see that to have an expression with a sedenion comprising 4 unit sedenions multiplied by another such, the resulting expression will have 16 product terms, for which pairwise cancellation is possible (16/2=8 new imaginaries introduced), so 4+4+8=16 independent imaginary components are needed (if we have enough imaginary terms to accommodate). This is not the case for sedenions but is the case for trigintaduonions. This exhausts the possibilities for sedenions, thus there are only the 84 sedenion zd's indicated. Continuing this analysis for trigintaduonions:

$$\{5\ \hat{e}_i\ term\} \times \{5\ \hat{e}_j\ term\} = \{25\ \hat{e}_j\ term\} \rightarrow odd\ terms, can't\ cancel\ pairwise, no\ zd's$$

$$\{6\ \hat{e}_i\ term\} \times \{6\ \hat{e}_j\ term\} = \{36\ \hat{e}_j\ term\} \rightarrow need\ 6 + 6 + 18 = 30\ indep. imag's$$

The latter case is still possible for trigintaduonions, which have 31 imaginary components.

Suppose for trigintaduonions instead of the antisymmetric tensor we have some more general third-rank tensor with no indices the same, again we will have a relation with the first two indices (ranging over sedenions), with two sign forms, giving rise to no more than 15*14*2=420 independent discrete trigintaduonion zero divisors. If we view the approximate number to be four times that of the sedenions since 1 T-multiplication can be turned into 4 S-multiplications we could argue for 4x84=336 trigintaduonion zero divisors. Regardless, it is a discrete set as with the Sedenions and that's all that's needed for the Methods to be discussed.

Integrals with large parameter

A review of oscillatory integrals now follows. This mathematics is critical to the path-integral quantization program. It traces to Laplace's method of steepest descents, then to the work of Stokes and Lord Kelvin, then to the work of Erdelyi and others [7-9], before its incorporation by Feynman into his path integral formulation of QM and QFT, where the most precisely tested result in physics was then shown with QED [10-12].

So far we've seen sums involving possibly an infinite number of products during an emanation step. These are related to a definite integral with appropriate measure. Thus we arrive at a discussion of definite integrals. Now if the integrand has maxima or stationary points the definite integral is often dominated by those regions (to be shown momentarily), so the focus turns to an asymptotic analysis of the integrals about those internal points (and boundary points), e.g, an asymptotic expansion analysis. (The easiest asymptotic expansion for a definite integral is obtained by repeated integration by parts.)

For what follows we are interested in the definite integral with large parameter:

$$f(x) = \int_a^b e^{xh(t)} dt$$

where x is very large (or grows large). Under these circumstances we are interested in any critical point t_0, where $h'(t_0) = 0$, and we write $h(t)$ in terms of its Taylor expansion about that critical point:

$$f(x) \approx e^{xh(t_0)} \int_a^b e^{-\frac{1}{2}x|h''(t_0)|(t-t_0)^2} dt$$

and we then approximate with the integration bounds at infinity to get the standard Gaussian integral. Thus, we get the asymptotic expansion solution for large x:

$$f(x) \approx e^{xh(t_0)} \sqrt{\frac{2\pi}{x|h''(t_0)|}} \quad for\ large\ x.$$

Let's now consider the more general form originally studied by Laplace:

$$f(x) = \int_a^b g(t) e^{xh(t)} dt$$

similar analysis can proceed if we make the substitution $h(a) - h(t) = u^2$ (due to Laplace), where a factor of $\frac{1}{2}$ is introduced since the eventual domain of integration will be $\{0,\infty\}$ not $\{-\infty, \infty\}$:

$$f(x) \approx g(t_0) e^{xh(t_0)} \sqrt{\frac{-\pi}{2xh''(t_0)}} \quad for\ large\ x.$$

If we consider the last integral but generalized to x a large complex variable, and g and h to be analytic functions of complex t (first studied by Riemann and Debye [9]), we have similar analysis but where we start by deforming the path of integration as much as possible to coincide with paths of steepest descent. What we seek, however, is not the full complex generalization, but the alternate form that might be arrived at by analytic continuation from the initial (pure) real form to a pure imaginary form (via a Wick rotation). For this we arrive at the integral:

$$f(x) = \int_a^b g(t) e^{ixh(t)} dt$$

where x is large and positive and $h(t)$ is real as before (but with the factor of i, now effectively pure imaginary). To solve this type of integral the integral is dominated by terms not cancelled, i.e., where the phase is stationary in the integration. This occurs at the critical points (and end-points) as before, but represents a slower convergence or domination about the critical point than that in the exponential fall-off case dealt with by Laplace. The method of stationary phase was initially developed by Stokes and Kelvin [7]. By similar arguments to that shown above we then arrive at the solution:

$$f(x) \approx g(t_0) e^{[ixh(t_0) + \frac{i\pi}{4}]} \sqrt{\frac{2\pi}{xh''(t_0)}} \quad for\ large\ x.$$

In the result above, the integral is dominated by the region around the stationary point. Since this integral is written $\{-\varepsilon, +\varepsilon\}$, which is extended to $\{-\infty, \infty\}$, we get the standard Gaussian integral factor. If we generalize to g and h analytic, then the point of stationary phase is a saddle-point in the complex plane and using methods like in the method of steepest descent, the integration path is deformed to traverse the stationary-phase saddle-points (from stirrup to stirrup) such that the same result above is obtained as the dominant contribution as x grows large [9].

Methods

Trigintaduonion Emanation: achirality from sum on all chirality

If analyticity confers Laplace's equation, from electrostatics for example, what may confer electrodynamics? For this we need something that can be dynamical and in the current theory of trigintaduonion emanation (projection) we only discussed how to connect to a 10D emanation with alpha perturbation into 32D, now complexified to 64D (surmised to have been there in the projection to the perturbed 32D state at the outset, with the initial trigintaduonion projection emergence). In practice, there are four chiralities, and for a given chirality (with unit norm) there are 29 dimensions of freedom (10D + 19D of chirally consistent perturbation). When analytic extension is taken to give maximal information flow, the effective dimension for each of the four chiralities is 29* (detailed in [4]). This clear decomposition into 29* independent effective dimensions is then revealed in the $\{\alpha,\pi,c_\infty\}$ relation in [4]. The Mandelbrot Set is one of many that encounter the universal constant c_∞. The Mandelbrot set also describes a 2D fractal boundary at its "edge of chaos". If driven to similar optimality in approaching a zero-value (a zero-divider issue), we see a two-value zero-crossing specification effectively like a double zero. The parameterization of the zeros of the Emanator at chiral zero-divisor points will thus be as double-zeros.

Recall the description of the emanator from [4]:

$$T_{chiral}^{(k)} = \begin{cases} ((0,\alpha),\beta) \\ ((\alpha,0),\beta) \\ (\beta,(0,\alpha)) \\ (\beta,(\alpha,0)) \end{cases}, where\ T_{chiral}^{(k)} = \mathbf{1} + i\boldsymbol{\delta}.$$

$$\text{Emanation}(\mathbf{T}) = \frac{1}{N}\sum_{k\in\{4x72\}^n} \mathbf{T} \bullet \boldsymbol{T}_{\boldsymbol{chiral}}^{(\boldsymbol{k})} = \frac{1}{N}\sum_{K\in 4\ chiralities} \mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}^{(\boldsymbol{K})}$$

Suppose we add the rule that emanation may not proceed when a particular chirality is zeroed-out, in other words:

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}^{(\boldsymbol{K})} \neq \mathbf{0}.$$

For 'normal' numbers this goes without saying, since for real numbers if we have $r_1 \times r_2 = r_3$ then $r_3 \neq 0\ if\ neither\ r_1 = 0\ or\ r_2 = 0$. This holds true for the Real, Complex, Quaternion, and Octonion numbers. This does not hold true for Sedenions or higher. For sedenions the dimensionality of the zero-divisor event is mostly constrained, while for trigintaduonions it is significant (see Discussion). If such zeros were eliminated from the emanator description by using analytic extension component-wise (on 29* effective components) we see how a description devoid of matter (pure static field with no source or sink) might acquire matter by way of extending to a maximal domain of analyticity be removing zero-divisor events (a Wick transformation from real dimensionless action to pure imaginary action that is dimensionless but consisting of a dimensionful ratio). For what follows, let's parameterize the zero-divisors and index them such that:

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}_{chiral}(\boldsymbol{S}^{*}{}_{i}) \to \mathbf{0} \; as\ double\ zero\ \forall \boldsymbol{S}^{*}{}_{i} \, .$$

Maximal emanator analyticity via removal of zeros

The sum over the zero-divisors means that the part of the emanator requiring analytic 'repair is given by:

$$\sum_{\{\boldsymbol{S}^{*}{}_{i}\}} \mathbf{T} \bullet \overline{\boldsymbol{T}}_{chiral}(\boldsymbol{S}^{*}{}_{i}) \to \boldsymbol{i} \sum_{\{\boldsymbol{i}\}} \boldsymbol{e}^{\boldsymbol{S}^{*}{}_{i}/\boldsymbol{h}^{*}} \, .$$

Where use is made of the fact that approach to zero-divisor (ZD) is purely involving imaginary components. The shift to exponential form will be explained with the choice of analytic continuation or 'repair' described in the next section. The sum on ZD events (for all 'time') can thus be described as a sum on (ZD) paths. The dimensionality of possible ZD's (for trigintaduonions) thus indicates a dimensionality on possible paths, with result:

$$\sum_{\boldsymbol{zd's}} \boldsymbol{e}^{\boldsymbol{S}^{*}{}_{i}/\boldsymbol{h}^{*}} \to \int_{\boldsymbol{zd\ paths}} \boldsymbol{e}^{\boldsymbol{S}^{*}(\boldsymbol{i})/\boldsymbol{h}^{*}} \, .$$

We can see now the identification of matter with the zero-divisor 'residues' that occur when imposing maximal analyticity. If we now do a Wick rotation and go from real dimensionless iteration-number to imaginary dimensionful action, with dimensionful Planck's constant. We then get the highly oscillatory integral that is the basis of quantum field theory and quantum mechanics, with their classical and semiclassical reductions. So, we go from an integral on zd paths with large parameter 1/h* to an integral on matter paths with large parameter 1/h. We, thus, maintain the large-parameter form as we go from a Laplace-type integral to a Stokes-type integral, and thus arrive at a path integral formulation:

$$\int_{\boldsymbol{zd\ paths}} \boldsymbol{e}^{\boldsymbol{S}^{*}(\boldsymbol{i})/\boldsymbol{h}^{*}} \to \int_{\boldsymbol{matter\ paths}} \boldsymbol{e}^{\boldsymbol{i}\boldsymbol{S}(\boldsymbol{i})/\boldsymbol{h}}, \qquad where\ \boldsymbol{S}(\boldsymbol{i}) = \int \boldsymbol{Ldt} \, .$$

Zero-divisor removal at component level

In the Results we will need zero removal for analyticity on the log of the trigintaduonion products for a particular chirality of emanation. Let's now calculate the zero removal residue seen as a product of each of the d-dimension number of analytically-continued real components. Recall that:

$$\oint_C \frac{1}{z} dz = \oint_C d(\ln z) = 2\pi i \quad (simple\ pole).$$

on a contour that encloses the pole, which generalizes to:

$$\oint_C d(\ln f(z)) = \sum_{zeros} 2\pi m i \quad (multiple\ zeros),$$

where f has multiple zeros of order m, and where the last result requires that $f(z)$ be analytic throughout the domain, D, with boundary C inside that analytic domain (and D is simply connected). Let $\boldsymbol{S^*}_{\boldsymbol{i}}$ be the zeros of $f(z)$ where at lowest order $f(z)$ has a double zero at each of the $\boldsymbol{S^*}_{\boldsymbol{i}}$ according to the max fractal dimension possible for the boundary condition at the edge-of-chaos (where the dim=2 boundary dimension is actually the case for the Mandelbrot Set [5]). Let's use this information to parameterize the approach to the zeros:

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{z}) \propto \prod_{\boldsymbol{d=29*}} (\boldsymbol{z} - \boldsymbol{S^*}_{\boldsymbol{i}})^{\boldsymbol{2}},$$

thus, for multiple zeros:

$$\oint_C \frac{d}{dz}(\ln[\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{z})])dz = \sum_{zeros} \prod_{29*} 4\pi i.$$

Focusing on just one of the zeroes and the line integral dominated by a local, stationary phase, contribution, we need to integrate and set $\boldsymbol{z} = \boldsymbol{S^*}_{\boldsymbol{i}}$:

$$d\left(\ln\left[\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{z})\right]\right) = 4\pi i^{29*} dz.$$

and, with choice of integration constants (phase factors):

$$\mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{S^*}_{\boldsymbol{i}}) = \boldsymbol{e}^{\boldsymbol{S^*}_{\boldsymbol{i}}/|\boldsymbol{h^*}|}.$$

Summing on the zeros of the latter expression:

$$\sum_{zeros\ \boldsymbol{S^*}_{\boldsymbol{i}}} \mathbf{T} \bullet \overline{\boldsymbol{T}}_{\boldsymbol{chiral}}(\boldsymbol{S^*}_{\boldsymbol{i}}) = \sum_{zeros\ \boldsymbol{S^*}_{\boldsymbol{i}}} \boldsymbol{e}^{\boldsymbol{S^*}_{\boldsymbol{i}}/|\boldsymbol{h^*}|}.$$

Thus, the general form of maximal, analytic, information emanation gives rise to a sum on residue-like terms associated with each of the zero-divisors (zd's), and an 'action' variable is indicated to result from the parameterization of the approach to each of the zero-divisors, with their individual actions additive (phase contributions multiplicative) for parts contributing to a particular zd. The sum over all the zd's will, upon analytic continuation, be associated with a sum over paths. The zd action variable is written in the form of the integral of a functional along a path parameterized by 'time', with the usual definition for Action if the functional is the Lagrangian:

$$\sum_{zeros\ S^*_i} e^{S^*_i/|h^*|} \rightarrow \int_{matter\ paths} e^{iS(i)/h}\ , where\ S(i) = \int Ldt\ ,$$

where the definition for action above is kept to the simple form for a point particle trajectory. More complex forms can be written for field descriptions, where we generalize from point particle forms in various ways, but still with point-like coupling terms. Further generalization to actions describing 1-D objects, string not points, or beyond (n-D objects, or branes) and their trajectories is possible at this point but note how the chain of associations is altered, if not broken. Tracing back to a fundamental issue of analyticity when going from emanator form to propagator form, we saw that analyticity requires *isolated* zeros to not make the entire solution trivially zero. Thus, the fundamental meromorphic 'precipitate' for matter might be point and point-based field constructs as we've developed them, where the role of string theory will emerge separately, although possibly as early as the "matter-precipitating" Wick rotation step above, with an appropriate density of zd's along a 1-D or n-D curve. (If nothing else, String theory provides a critical QG renormalization construct, emergent with the spacetime geometry itself.) Note that in going from

$$\frac{S^*_i}{|h^*|} \rightarrow i\frac{S(i)}{h}$$

(1) Both ratios are dimensionless, but the quantities on the left are a ratio of pure numbers themselves dimensionless, while the RHS has a ratio of S, the action, with action dimensions, and h, Planck's constant, also with action unit dimensions.
(2) Both $1/|h^*|$ and "$1/|h|$", where the abs value operation on the latter simply drops the dimensionful units, are extremely large numbers, and for the latter, occurring in a phase argument, this *sets up a highly oscillatory integral* such that the classical solution $\delta S = 0$ results, (if a classical solution exists for the system studied), among other things.

Any meromorphic function on a sphere, due to its compactness, must be rational. Evaluation of zero divisors occurs when we consider the product of two pure unit-norm imaginary trigintaduonions, thus we consider analyticity on a 31-sphere with its possible decomposition into analytic extensions on 31 real trigintaduonion components into 31 complex trigintaduonion components.

If the emanator is to remain an achiral mix, as well as analytic, then we can't allow the zero divisor events that would drop a chirality mentioned above, where these occurrences are treated as isolated singular events removable from the domain of definition of the emanator by repeated application of the analytic domain 'surgery' (repeated on both events, and for given event, it's different independent components). This analytic 'surgery' occurs for each of the independent component dimensions for a given chiral emanation, and for each of those dimensions it returns a zero-removal 'residue' of $4\pi i$ (with an extra factor of 2 since a double zero at the fractal boundary). We found in [4] that the effective dimension is 29*, thus the remnant of the surgery for each zd removed is:

$$1/|h^*| = (4\pi)^{29^*}$$

A quantization on the matter has occurred thus far in the sense that the meromorphic function must be rational, so a discrete, countable, number of matter-associated events must occur.

Note that we have:

$$|\boldsymbol{h}^{*}| = \boldsymbol{6.630 \times 10^{-33}} \qquad \boldsymbol{vs} \qquad \boldsymbol{h = 6.626070 \times 10^{-34} J\ s}$$

where we only need these two 'h' numbers to satisfy the same extreme smallness property in order to obtain integrals with large parameter and thus *a highly oscillatory integral with stationary phase domination*.

Achieving Dimensionality

Unlike Boltzmann's constant, which can be eliminated by appropriate choice of units, here we cannot eliminate the dimensionful part of Plank's constant (e.g. "$J\ s$"), although a slight shift to a time unit of ~1/10 second, a "decond", or "dec" or"d", say, could lead to:

$$\boldsymbol{h = |h^{*}|\ J\ d}$$

What remains is still the problem of how to dress up the key parameters with dimensionful units, to arrive at the standard physics formulations, when the original formalism is purely algebraic, albeit with dimensionless constants already found to exist (alpha). This is accomplished by the Wick rotation from integration on real terms to integration on imaginary phase contributions. In making this analytic continuation we introduce units via transforming a dimensionless ratio of dimensionless numbers to a dimensionless ratio of *dimensionful* numbers. We also go from summing on zero-divisor associated terms to summing on zero-divisor associated 'paths'. The summations on path add according to their phase, the latter dependent on the action expressed as

$$\boldsymbol{S = \int Ldt}$$

where time emerges as the parameterization of the path. Analyticity on this integral (and all integrals encountered thus far), in the form of the Wick rotation especially, is what is referred to as Euclideanization in later sections. Note, this describes a doubly analytic structure (at level of emanator and at level of propagator) just as there was a doubly chiral extension (for maximal emanator). Since the Wick rotation on the trigintaduonion (32D) objects represents use of an analytic complex structure to extend each of the real components to complex components, we have an analytic extension off of the 32D Cayley algebra into the enveloping 64D Cayley algebra. Perhaps a better way to view this is that the emanation process that arrived at the 32D Cayley algebra did so in a context where an analytic 64D Cayley algebra extension already existed.

For a zero-divisor to occur with the Trigintaduonions the real component must be zero, but this is possible for the base trigintaduonion in the Emanator (the number of emanation steps to a zero-crossing event, with random-walk statistics on the real component, is examined in [3]). We now continue the discussion of zero divisors with a description of new results.

Results

Emanation when base trigintaduonion contains Zero Divisors

Consider emanation when the base trigintaduonion is a zero-divisor $\mathbf{T}_{\boldsymbol{ZD}}$:

$$\text{Emanation}(\mathbf{T}_{\boldsymbol{ZD}}) = \frac{1}{N} \sum_{k\in\{4x72\}^n} \mathbf{T}_{\boldsymbol{ZD}} \bullet \boldsymbol{T}_{\boldsymbol{chiral}}^{(\boldsymbol{k})},$$

and suppose the number n (like the number of cards in 'flop' to make a reading) is such that $\{4x72\}^n$ is large, such that the sum on trigintaduonion products is dominated by stationary phase terms. Such domination by stationary phase is expected with appropriate handling on the normalization, even without zero real component and unit norm, since we have phase addition on a compact space, the 31-sphere. We now have a new mechanism driving the stationary phase solution, however, due to the existence of zero divisors, for which a new type of solution class is indicated. Suppose stationary phase in this context selects such that:

$$\text{Emanation}(\mathbf{T}_{\boldsymbol{ZD}}) = \frac{1}{N}\mathbf{T}_{\boldsymbol{ZD}}\bullet(\mathrm{R} + \mathbf{T}_{\boldsymbol{ZD}*}) = \mathbf{T}_{\boldsymbol{ZD}}, \qquad \Delta\mathbf{T}_{\boldsymbol{base}} = \mathbf{0}$$

where $\mathbf{T}_{\boldsymbol{ZD}*}$ is the zd conjugate to $\mathbf{T}_{\boldsymbol{ZD}}$, i.e. $\mathbf{T}_{\boldsymbol{ZD}}\bullet\mathbf{T}_{\boldsymbol{ZD}*} = \mathbf{0}$**,** and N is the appropriate normalization constant to arrive at unit norm as before. Since $\Delta\mathbf{T}_{\boldsymbol{base}} = \mathbf{0}$, in the emanation process it is unchanging, thus this is the condition that will relate to the classic equilibrium (or stationarity at least).

Let's now consider the $\mathbf{T}_{\boldsymbol{base}}$ that consists of a sum over a countable collection of zero divisors with separate weighting factors:

$$\mathbf{T}_{\boldsymbol{base}} = \sum_{i\,\epsilon\,all} a_i \mathbf{T}_{\boldsymbol{ZD,i}}$$

Suppose stationary phase in this context selects such that:

$$\text{Emanation}(\mathbf{T}_{\boldsymbol{base}}) = \frac{1}{N}\mathbf{T}_{\boldsymbol{base}}\bullet \sum_{i\,\epsilon\,all} \mathbf{T}_{\boldsymbol{ZD*,i}} \bullet(\mathbf{T}_{\boldsymbol{ZD*,i}})^{-\mathbf{1}} = \mathbf{T}_{\boldsymbol{base}}$$

where the order of 3-T multiplications is with the inverse last, and where an overall constant is eliminated by the renormalization term to arrive back at the starting base trigintaduonion. This appears to be the general condition for describing the emanation form of equilibrium. Let's now consider what happens if the real component is nonzero as well (and assume 420 ZD's):

$$\mathbf{T}_{\boldsymbol{base}} = \boldsymbol{R} + \sum_{i\,\epsilon\,all} a_i \mathbf{T}_{\boldsymbol{ZD,i}}$$

$$\text{Emanation}(\mathbf{T}_{base}) = \frac{1}{N}\Big(\boldsymbol{R} + \sum_{i\,\epsilon\, all} a_i \mathbf{T}_{ZD,i}\Big)\bullet \sum_{i\,\epsilon\, all} \mathbf{T}_{ZD*,i} \bullet (\mathbf{T}_{ZD*,i})^{-1}$$

$$\text{Em}(\mathbf{T}_{base}) = \frac{1}{N}\left(420\boldsymbol{R} + \sum_{i\,\epsilon\, all} 419 a_i \mathbf{T}_{ZD,i}\right) = \frac{\mathbf{1}}{\mathbf{N}}(\mathbf{R} + \mathbf{419T}_{base}) \cong \mathbf{T}_{base}$$

with a slight overall increase in the real component, and notably retaining all of the ZD's.

Let's now consider the general case where ZD's are indicated as a separate portion (and assume 420 ZD's):

$$\mathbf{T}_{base} = \left(\boldsymbol{R} + \boldsymbol{T}_{imag}\right) + \sum_{i\,\epsilon\, all} a_i \mathbf{T}_{ZD,i}$$

and

$$\text{Em}(\mathbf{T}_{base}) = \frac{\mathbf{1}}{\mathbf{N}}\left(\left(\boldsymbol{R} + \boldsymbol{T}_{imag}\right) + \mathbf{419T}_{base}\right) \cong \mathbf{T}_{base}$$

with a slight overall increase in the non-ZD part while still notably retaining all of the ZD's. There is thus conservation of ZD's, suggesting association of ZD's with matter/energy and the conservation of the latter seen in the emanated propagator formalism. The nature of this matter association is still unclear, however, until we consider the next condition on the emanator.

Let's now consider the form of the emanator when it is summed into the 4 chiralities (or 78 or 72 card types dependent on form):

$$\text{Emanation}(\mathbf{T}_{base}) = \frac{1}{N} \sum_{K \in 4\ chiralities} \mathbf{T}_{base} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{chiral}}$$

and, thus

$$\mathbf{T}_{base} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{chiral}} \neq \mathbf{0}$$

In this context the zero divisors in the base force an unexpected constraint if we require that no elimination of chirality (thus violation of emanator achirality) can occur. In other words, we hypothesize the emanation is constrained such that it is analytic on the log of the products such that zero's are eliminated from the maximal analytic domain.

On the other hand, suppose the form of the emanator can be written as a sum on achiral groups. Such groups *can* be zeroed-out, which describes a form of wave-collapse or measurement filter for the theory:

$$\text{Emanation}(\mathbf{T}_{base}) = \frac{1}{N} \sum_{K \in achiral\ group} \mathbf{T}_{base} \bullet \overline{\boldsymbol{T}}^{(\boldsymbol{K})}_{\boldsymbol{achiral}}$$

and, thus, we can have:

$$\mathbf{T}_{base} \bullet \overline{\boldsymbol{T}}_{achiral}^{(K)} = \mathbf{0}.$$

From the preceding results we then see that we can formulate a hypothesis for the meromorphic precipitation of quantum matter with dimensionful action, where:

(1) The trigintaduonion emanator is doubly analytic, where the first analyticity is in regards to removing the zero-divisors from the domain of the trigintaduonion emanator by means of analytic operations to remove the zero-event for each of the effective dimensions, giving rise to a dimensionless 'action' $\boldsymbol{S}^*$ and a quantum of that action given by:

$$|\boldsymbol{h}^*| = \left(\frac{\mathbf{1}}{\mathbf{2}\boldsymbol{\pi m}}\right)^{\mathbf{29}^*}, where\ \boldsymbol{m} = \mathbf{2}.$$

While the second analyticity is in regards to the resulting sum on associated zero-divisor paths. Upon analytic operation (Wick rotation) we arrive at a sum on paths whose phase is given by a dimensionful action with respect to a dimensionful quantum of action (Planck's constant):

$$\boldsymbol{S}^*/\boldsymbol{h}^* \rightarrow \boldsymbol{S}/\boldsymbol{h}$$

(2) We arrive at large-parameter integral over paths, that is highly oscillatory given $|\boldsymbol{h}^*| \approx |\boldsymbol{h}| \ll \propto < \mathbf{1}$, and it must satisfy the quantum deFinetti relation [13], to give rise to a real action, with:

$$\boldsymbol{S} = \int \boldsymbol{Ldt}$$

where the real-valued Lagrangian is selected to be at a variational optimum.

Discussion

We have 6-element, 4-element, and 2-element antisymmetric zero-divisors that are summed in the same emanator expression, giving 3 independent 'coordinates' to a zero-divisor event, and thus of matter. The collection of the matter part of the emanator is pure imaginary. The non-matter part (real and some imaginary) is increasing when no new zd's are created. Thus, we see a local-time notion as well as a preservation of the mix of zd's but with them being less of the whole – i.e., inflation.

We see matter as meromorphic residue precipitation, in amounts of one quantum given by a precursor to Planck's constant h*. The meromorphic residue winding number is also notable in that it gives an integer that stays constant in the meromorphic region. This raises the possibility that elementary particle attributes might encode by way of different analytic extensions (complex structures), with reference to their different winding numbers at residues, but that will not be discussed further here.

We know from [3] that the chiral trigintaduonion emanation theory indicates 22 free parameters with maximum perturbation amount alpha in the larger 32D trigintaduonion algebra. In the analysis of the possible emanators *analyticity* is indicated in numerous ways, such that this is a core hypothesis for the maximal information propagating solution. This, in turn, indicates analytic surgery via the residue theorem, on the log of the emanator, to create a maximal analytic region. When we Wick rotate from $\boldsymbol{S}^*/\boldsymbol{h}^* \rightarrow \boldsymbol{S}/\boldsymbol{h}$, there should be 22 independent parameters in the action $\boldsymbol{S}$ [3], with Plank's constant counted separately. Can we fit the parameters of the Standard Model, a possible extension for dark matter (e.g., neutrinos with possible exotic effect), and the gravitational constant G all into that 22 count? Yes, if we adopt the Koide relation [14]. Let's show this by first listing the 19 parameters in the Standard Model:

(I) 9 Yukawa coupling constants (masses) for the charged fermions
(II) 5 constants for Weinberg Angle and the CKM matrix (with three mixing angles and CP-violating phase)
(III) 3 Constants for electromagnetic coupling (alpha), for strong interaction (g3), and strong CP-violating phase ($\theta_3 \approx 0$).
(IV) 2 Higgs parameters: Mass and Vacuum Expectation

If we allow for the neutrinos to have mass, then we get 3 more masses and another 4 constants for the PMNS matrix (three mixing angles and a CP-violating phase):

(V) Extended model: 7 more constants → We, thus, have 26 parameters.

If we add the constant for Gravitation (G) to have all constants for Std. Model + Gravitation, we now have 27 parameters. Note, however, that the alpha constant is listed above as the EM coupling constant, but isn't really a separate parameter since it is the same for any emergent chiral trigintaduonion emanation. This is all the more apparent if we go with a listing of 19 independent parameters in terms of the g_1 and g_2 coupling constants which share the following relation with alpha:

$$\alpha = \frac{1}{4\pi} \frac{{g_1}^2 {g_2}^2}{{g_1}^2 + {g_2}^2}$$

So, we take alpha away from the count to get to 26. This is where the Koide relation comes into play.

The Koide relation [14] was first observed for the three massive leptons currently known:

$$\frac{m_e + m_\mu + m_\tau}{\left(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau}\right)^2} = \frac{2}{3}$$

To a lesser extend this relation is satisfied for the quarks as well, particularly for the three most massive, where the value is 0.6695. The problem with a simple application to the quark masses is that they are dependent on energy scale. A theoretical explanation for the Koide relation describes how this relation might exist for the masses of a given generation (or family group) [15]. Assuming this or some other theoretical explanation can show that the three masses of a given generation aren't truly three independent parameters, but two. With this correction on 4 generation of masses (now counting the neutrino generation), we arrive at 26-4=22 free parameters as desired., and the emanator theory thus indicates a nearly complete theory in that the 22 parameters are almost known.

Conclusion
The fine-structure constant alpha and Planck's constant have very different trigintaduonion emanation origins and uniqueness:

1. Alpha derives from T-emanation directly, without reference to zero divisors, is dimensionless, and is precisely defined.
2. Planck's constant is only partly specified, with its essentially small quantum, to establish an oscillatory integral with h*<<alpha, which derives from T-emanation when zero divisors are accounted for by way of maximal analyticity.

References

1. Winters-Hilt, S. Feynman-Cayley Path Integrals select Chiral Bi-Sedenions with 10-dimensional space-time propagation, Advanced Studies in Theoretical Physics, Vol. 9, 2015, no. 14, 667-683 (2015).

2. Winters-Hilt, S. Unified propagator theory and a non-experimental derivation for the fine-structure constant. Advanced Studies in Theoretical Physics, Vol. 12, 2018, no. 5, 243-255. https://doi.org/10.12988/astp.2018.8626.

3. Winters-Hilt, S. The 22 letters of reality: chiral bisedenion properties for maximal information propagation. Advanced Studies in Theoretical Physics, Vol. 12, 2018, no. 7, 301-318. https://doi.org/10.12988/astp.2018.8832.

4. Winters-Hilt, S. Fiat Numero: Trigintaduonion Emanation Theory and its Relation to the Fine-Structure Constant α, the Feigenbaum Constant c_∞, and π. Advanced Studies in Theoretical Physics Vol. 15, 2021, no. 2, 71 - 98 HIKARI Ltd, www.m-hikari.com https://doi.org/10.12988/astp.2021.91517.

5. Shishikura, Mitsuhiro (1998). "The Hausdorff dimension of the boundary of the Mandelbrot set and Julia sets". Annals of Mathematics. Second Series. 147 (2): 225–267.

6. https://en.wikipedia.org/wiki/Sedenion

7. Erdeyli, A. Asymptotic Expansions. 1956 Dover.

8. Erdeyli, A. Asymptotic Expansions of differential equations with turning points. Review of the Literature. Technical Report 1, Contract Nonr-220(11). Reference no. NR 043-121. Department of Mathematics, California Institute of Technology, 1953.

9. Carrier, G.F, M. Crook and C.E. Pearson. Functions of a complex variable. 1983 Hod Books.

10. Feynman, R.P. Space-Time Approach to Non-Relativistic Quantum Mechanics. Rev. Mod. Phys. 20, 367 – Published 1 April 1948

11. Feynman, R.P. (1950) Mathematical Formulation of the Quantum Theory of Electromagnetic Interaction. Physical Review, 80, 440-457. https://doi.org/10.1103/PhysRev.80.440

12 Feynman, R.P. and Hibbs, A.R. (1965) Quantum Mechanics and Path-Integral. McGraw-Hill, New York.13. Caves

14. Koide, Y., Nuovo Cim. A 70 (1982) 411 [Erratum-ibid. A 73 (1983) 327].

15. Sumino, Y. (2009). "Family Gauge Symmetry as an Origin of Koide's Mass Formula and Charged Lepton Spectrum". Journal of High Energy Physics. 2009 (5): 75. arXiv:0812.2103.